Wavelet Leader and Multifractal detrended uctuation analysis of market effciency : Evidence from WAEMU market index

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Keywords: Efficient market hypothesis, MF-DFA, Wavelet, Hurst exponent, BRVM10

Wavelet Leader and Multifractal detrended fluctuation analysis of market efficiency : Evidence from WAEMU market index

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Abstract

The dimension of efficiency is of main importance in the financial markets, (Fama, 1965) considered as the father of this theory has defined the information efficiency in three forms (weak, semi-strong and strong). It is for this reason that the information finds its importance in this theory because the nature of the information (past, public, private) makes it possible to determine the form of efficiency. However, this theory of efficiency has long been a controversial and unresolved issue in the economic literature. Most of the work is done on developed country stock exchanges. This problematic of informational market efficiency is a current topic and has been the subject of much study with different methods. In this paper we analyze the hypothesis of weak form informational efficiency of the market. Our empirical study focuses on the BRVM10 index of the WAEMU regional stock exchange over the period from 17/04/2000 to 23/08/2016. In this paper, we use wavelet approach and multifractal detrended fluctuation analysis (MF-DFA). The study revealed that the log returns of the index exhibits a persistent and multifractal procesus.

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JEL Classification: C02, C4, C12, G1, G14

1. Introduction

The many theoretical and empirical literature analyzes the behavior of the stock price reveals that the stock price is not only a value observed at the moment but contains all the information available

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on the market. This concept makes use of the efficient market hypothesis notion. Since the late 1950s, most of the works have been consulted as part of the financial markets efficiency assumption. Thanks to her, finance has made great progress. This is a subject that has led the production of rich and diverse works of great researchers. It has given rise to a very important revision with different dimensions according to (Kabbaj, 2008), one can distinguish the allocative efficiency [1] , the operational efficiency [2], the informational efficiency. The informational efficient market hypothesis is derived from the work of Cootner [1964]. Despite previous work, the paternity of this theory is generally attributed to Fama thanks to his thesis defended in 1965, he published an article in the Journal of Finance entitled "Efficient Capital Markets". (Fama, 1965) has defined an efficient market as the one on which prices reflect the available information at all times. Always in the same direction (Fama, 1970) classified efficiency in three forms according to information set. These include the weak, semi-strong and strong forms. The study of the behavior of share prices is of great importance from the point of view of the policies. And any kind of extraordinary or random movement in prices that is grossly out of line with economic fundamentals raises concerns for both market practitioners and policymakers alike. As a result, the understanding and analysis of stock price behavior has interested academics in general and modelers in particular. The efficient hypothesis of financial markets has certainly made enormous progress with academics and professionals thanks to the different approaches proposed. Only, the majority of studies carried out concern the developed stock markets. With the advent of certain phenomenon such as globalization or financial integration, the gaze is increasingly focused on the emerging or underdeveloped markets (Feldstein, 2000),(Stulz, 1999),(Stiglitz, 2005),(Claessens, et al.,2001),(Chinn and Ito, 2005) However, we must note the frequency of certain events such as the shocks or crisis market (stock market crash (1929), crash of October 1987 and the second half of 1997, the bubble 2000, the subprime crisis 2008 ...). These facts revive debates on informational efficiency (Urrutia, 1995), (Mignon, 1998),(Colmant et al., 2009), (Gillet and Szafard., 2004),(Mignon, et al.,2006), (Khamis et al., 2018). One can wonders even if the repetitive crashes are not related to a problem of transparency or information utilisation in the financial markets? Thus, the problem of transparency on the financial markets and its utilistion is today at the core of debates between economists. In other words, the problems associated with the information disseminated at the stock exchange is a current topic that provokes debate. Therefore, the questions revolves more around these reports to know if these repetitive crises can not be factors of questioning the efficient hypothesis of financial markets? This issue preoccupied today to all of financial market participants. It is in this sense that we have access to our study on informational efficiency. This article focuses on the weak form efficiency of the BRVM10 index of the WAEMU regional stock exchange. Thus, we are particularly interested in the predictability of stock prices in this market. There exist a battery of test to examine the informational efficiency of the financial markets (random walk, autocorrelation test, unit root test, econometric models, etc.). However, it should be noted that these methods have neglected important factors: "information" and "time". We distinguish two schools to evaluate the share price; for

[1] The allocative efficiency is to allocate funds to the most productive jobs. Pareto [1909] states that a market is deemed to be optimal Pareto if it is possible to increase the welfare of an agent without injury to one or more other agents.

[2] Operational efficiency refers to the organization and the market structure. "The institutional, regulatory and technological characteristics, summarized under the name " microstructure of the markets ", significantly influence the securities supply and demand strategies of the various parties involved in the exchange process and, consequently, the prices training of assets listed". See (Gillet and Szafard., 2004)

chartists forecasts must be made from historical prices while fundamentalists consider that future prices can be only influenced by economic fundamentals. So in a financial, market operators can intervene for various horizons with a specific use of information. In the 1960s, Mandelbrot was the first to approach and showed the presence of scale laws in the markets. It highlights scale invariance (that is, a fractal market structure), which then allowed a market to be observed at any arbitrary scale as there was no more than privileged time to capture the fundamental behavioral structure of its fluctuations. All timescales were therefore suitable for statistical analysis. The fractal hypothesis of Mandelbrot drew attention to this problem and induced a new take into account of the scales. However, there is no particular reason for the variations corresponding to the short time horizon of the trader and those corresponding to the long horizon of the portfolio manager can be modeled by the same law of probability. This simultaneous coexistence of specific moments requires the instrumentation of analyzes and multi-scale markets (time of traders, time of leaders, time of insurers, etc.) to not lose information on the phenomenon studied. The debate around the modeling of fractal markets is precisely on this point and involves the laws of dynamic scales. (Levy, 1925),(Levy, 1937),(Mandelbrot, 1962),(Mandelbrot, 1963),(Fama, 1965),(Oswiecimka, et al,2005),(Wang, et al.,2009),(Yuan, et al 2009) . Which motivates the choice of this method. MF-DFA is a method for studying the multifractality of the market, measuring long-term dependency (LRD) from the Hurst exponent. The rest of the document is structured as follows. Section 2 reviews the existing literature of financial market efficiency hypothesis. Section 3 presents the methodology used. Section 4 examines the data and the results obtained. Section 5 presents the conclusions.

2. Literature review

As previously stated, the financial markets efficiency theory has attracted a lot of interest from the academic community, various tests categories were used. In weak sens, the predictability tests are generally used. Hence the utilisation of "random walk" is possible, an assumption that an analysis of past (current)prices can not allow a forecast present prices (future). The hypothesis confirmation may be favorable to efficiency in the weak form. First-order linear autocorrelation tests (that is, between t and $t-1$) very often led to results consistent with the random walk hypothesis. The random walk hypothesis is validated by some economists such as (Fama, 1965),(Working, 1934),(Kendall, et al.,1953),(Osborne, 1959),(Alexander, 1961),(Samuelson, 1965),(Hagin, 1966),(Niederhoffer et al., 1966),(Sharma and Kennedy, 1977),(LeRoy, 1973), (Lucas, et al.),(De Bondt, et al.,1985). While another group of authors rejects this hypothesis (Cootner, 1964), (Cowles et al., 1937). They argue that stock market price variations have some dependence, which has led to the publication of several books, including Cootner's (1964) collection of The Random Character of Stock Market Prices.

There are other tests in the literature that can test this hypothesis (the unit root tests, the variance ratio test, the run test, and the BDS test . . .). (Chowdhury, 1994),(Choudhry, 1994),(French and Roll., 1986),(Summers, 1986),(Poterba and Summers., 1988),(Lo and MacKinlay., 1988),(De Bondt, et al.,1985), (Urrutia, 1995),(Barnes, 1986),(Worthington and Higgs, 2006),(Sharma and Kennedy, 1977),(Seiler and Walter, 1997),(Ryoo and Smith, 2002),(Chang and Ting, 2000),(Myung, et al.,1991),(Maria, 2007),(Kwiatkowski et al., 1992),(Phillips and Perron., 1988) The studies on the co-integration relationship (Rush and Hakkio, 1989),(Mobarek, and Fiorante,2014). However, there is a new trend, including the utilization of wavelets and Multifractal Fluctuation Analysis (MF-DFA). These two approaches make it possible to take into account the share prices behavior at different time scales. Instead the classical traditional methods (Kumar and Kamaiah., 2014) used the wavelet method for study the weak form efficiency of the NASDAQ, DJIA and S & Poor indices (from 04-01-1980 to 12-09-2013). They used multi-scale entropy analysis by a MODWT decomposition and extract Sample entropy measure across different timescale. They find that markets are informational efficient in weak sense only in long term (semi-annual, annual). That is to say, as the time horizon increases, the markets evolve towards efficiency. (Simonsen et al., 1998) propose a multi-scale method, the Average Wavelet Coefficient Method (AWC) to compute the Hurst Exponent. The AWC method is multi-scales method, in sense where the behaviour at different scales does not influence each other in any significant way, i.e., the method decouples scales. (Fernandez-Martinez et al., 2016) propose a new method based on a multiscale lifting to estimate the Hurst exponent. The advantages of this approach to the existing Hurst parameter estimation is that, it naturally copes with data sampling irregularity. They shew that the virtually all existing Hurst parameter estimation methods which assume a regularly sampled time series and require modification to cope with irregularity or missing data which introduce higher estimator bias and variation. Many previous studies have demonstrated the use of the hurst exponent in the analyzing of weak form market efficiency (Pascoal, and Monteiro,2017),(Kumar and Kamaiah., 2014). (Wang, et al.,2009) using MF-DFA and divided their series in sub-series found that Shenzhen stock market was becoming more and more efficient by analyzing the change of Hurst exponent and a new efficient measure, which is equal to multifractality degree sometimes. They also shown that the volatility series still have significantly long-range dependence (LRD) and multifractality indicating that some conventional models such as GARCH and EGARCH cannot be used to forecast the volatilities of Shenzhen stock market. (Lahmiri, 2017) studies the multifractality of Moroccan family business stock. The results show that short (long) fluctuations in family business stock returns are less (more) persistent (anti-persistent) than short fluctuations in market indices. In addition, both serial correlation and distribution characteristics significantly influence the strength of the multifractal spectrums of CSE and family business stocks returns. Furthermore, results from multifractal spectrum analysis suggest that family business stocks are less risky. Thus, such differences in price dynamics could be exploited by investors and forecasters in active portfolio management. (Taro Ikeda, 2018), using the MF-DFA, shown that Russian stock market is characterized by a multifractal structure. They conclude that the multifractality degree of the Russian stock market can be categorized within emerging markets. (Suarez and Gomez, 2018) shew that the high-frequency returns of Madrid's Stock Exchange Ibex35 index exhibits a wide singularity spectrum which is most likely caused by its long-memory. (Tiwari et al., 2017),using Hurst exponent and multifractal detrended fluctuation analysis (MF-DFA) methods, has shown the mulfractality of this index. Another important results are the utilities and consumer goods sector ETF markets are more efficient compared with the financial and telecommunications sector ETF markets, in terms of price prediction, , there are noteworthy discrepancies in terms of market efficiency, between the short- and long-term horizons and the ETF market efficiency is considerably diminished after the global financial crisis. (Khamis et al., 2018) apply the MF-DFA approach to study the efficiency of Bitcoin market compared to gold, stock and foreign exchange markets. They found that the long-memory feature and multifractality of the Bitcoin market was stronger and Bitcoin was therefore more inefficient than the gold, stock and currency markets

3. Methodology

In this section we briefly present the methods of analysis used in the weak efficiency analysis: wavelet-based methods and the multifractal detrended fluctuation analysis method. The advantage on this approaches is that the wavelet method can eliminate some trend as result of vanishing moment property and the the MF-DFA method allows to avoid superiors detection of correlation that are artifacts of the non stationarity stock market index.

3.1. *Wavelet-based*

3.1.1. *Wavelet-based Hurst estimation*

Many authors have proposed the wavelet method for estimating the Hurst exponent (Abry, et al.,1998) and has been improved by (Simonsen et al., 1998),(Abry, et al.,2000),(Fernandez-Martinez et al., 2016),(Abry, et al.,2013). The advantage of this approach is it permit to capture the time varying proprieties of Long memory process and the self similarity caling behavior. (Abry, et al.,1998) show that the Hurst coefficient estimator obtained by the wavelet method is unbiased and efficient under certain general conditions.
Let be $X(t)t \in [0, n]$ a stationary series of second order spectrum Γ_ν that satisfies the following conditions

$$\Gamma_X(\nu) \sim C_X(\nu)^{-(2H-1)}; |\nu| \longrightarrow 0 \text{ and } \frac{1}{2} < H < 1 \tag{1}$$

Let be $\psi(t)$ the mother wavelet, the wavelet coefficients are obtained by:

$$d_X(j,k) = \int_{\mathbb{R}} \psi_{j,k}(t)x(t)dt \quad \text{où} \quad \psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t-k) \tag{2}$$

If $X(t)$ is stationnary

$$E_{d_X}(j,k) = \int_{\mathbb{R}} |\Gamma_X(\nu)\Gamma(2^j\nu|d\nu \tag{3}$$

Taking the density function property, the equation [3] becomes

$$E_{d_X}(j,k) \simeq C_X 2^{j(2H-1)} \; 2^j \longrightarrow \infty \tag{4}$$

By taking the log of equation [4] we find

$$\ln(E_{d_X}(j,k)) \sim j(2H-1) + \ln_2 C_X \quad \text{with } 2^j \longrightarrow \infty \tag{5}$$

By asking

$$S_j = \frac{1}{n_j}\sum_{k=1}^{n_j} d_X(j,k)^2 \tag{6}$$

with n_j the number of wavelet coefficients available at the 2^j scale, the estimate of $E(d_X((j,k))^2$ can be done using S_j by a weighted linear regression because the heteroscedasticity of $\log_2 S_j$. With

$$\ln_2(S_j) = (\ln_2 e)^2 C(j) \tag{7}$$

3.1.2. Wavelet Based multiscal diagram

The procedures for estimating the Hurst exponent (H) since the article by (Abry, et al.,1998) have undergone several extensions. We can thus have a H which depends on q ; $H(q)$, the case of multifractality. This method studies the variation of H using the multifractal spectrum of Legendre. This spectrum can be obtained by the partition function.

$$Sq(\tau) = \int |X(\omega, t+\tau) - X(\omega, t)|^q dt \tag{8}$$

with

$$\tau \longrightarrow 0 \ |X(\omega, t+\tau) - X(\omega, t)| = |\tau|^{H(\omega,t)} \tag{9}$$

Multifractal process ω :
Probability element
The transform of the Legendre function $\xi(q)$) by the Legendre function gives the multifractal spectrum of Legendre (Abry, et al.,2013) uses multi-scale diagrams to study the multifractal process The wavelet processes are an implement adapted to the calculation of $\zeta(q)$

$$\int \mid \Gamma_X(a,t) \mid^q dt = a^{\zeta(q)+1/2q} \quad \text{with } a \longrightarrow 0 \tag{10}$$

$$\text{where } T_X(a,t) = \int \frac{1}{\sqrt{a}} X(\omega) \psi \left(\frac{\mu - t}{a} \right) d\mu \tag{11}$$

According to the law-power

$$S(j,q) = \frac{1}{n_j} \sum_{k=1}^{n_j} \mid d_X(j,k) \mid^q \simeq 2^{j\zeta(q)} \in \mathbb{R} \text{ for small j} \tag{12}$$

We can thus estimate $\zeta(q)$ a linear regression. If $\zeta(q) = hq$, the obtained process is monofractal the spectrum is determined by H is a self-similar process and LRD. However, if $\zeta(q)$ is nonlinear we have a multifractal process.

3.2. Wavelet Leader for multifractal analysis

We describe in this subsection the procedure "wavelet leader for multifractal" Wendt (et al 2009). Let $\psi(t)$ be a compact time support mother wavelet function and N vanish moment Let $\lambda_{j,k} = \left[k2^j \ (k+1)2^j\right[$

a dyadic interval and $3\lambda_{j,k}$ the interval defined by

$$3\lambda_{j,k} = \lambda_{j,k-1} \cup \lambda_{j,k} \cup \lambda_{j,k+1} \tag{13}$$

The wavelet Leader is defined as the local supremum of the wavelet coefficients taken within a spatial neighborhood over all finer scale, (Jaffard, et al.,2005),(Wendt, et al 2007):

$$L_x(j,k) = \sup_{\lambda' \cup 3\lambda} |d'_\lambda| \tag{14}$$

$L_x(j,k)$ consists of largest wavelet coefficient $d_x(j'k')$ calculated at all finer scales $2^{j'} \geq 2^j$ within a narrow time neighborhood $(k-1)2^j \geq 2^j < (k+2)2^j$

3.2.1. Wavelet Leader multifractality formalism

"The wavelet leader multifractal formalism" allows to estimate Dh from the defined structure function:

$$S(q,j) = \frac{1}{n_j} \sum_{k=1}^{n_j} L_x^q\,(j,k) \tag{15}$$

Where n_j is the number of Leaders available at scale 2^j is q-moment of $L_X(j,k)$ at scale. Assuming structural functions Behave at Scale 2^j

$$S\,(q,j) \simeq C^L(q) 2^{j\zeta(q)} \tag{16}$$

A Legendre transformation of scale exponent $\zeta(q)$ which allows an estimation of multifractal spectrum, (Jaffard, et al.,2005)

$$\mathfrak{L}(h) = inf_q\,(1 + qh - \zeta(q)) \leq D(h) \tag{17}$$

where

$$\zeta(q) \triangleq \lim_{j\to\infty} inf \frac{\log_2 S(q,j)}{-j} \tag{18}$$

$S(q,j) = 2^{-j\zeta(q)}\ j \longrightarrow +\infty$

In addition the structure function can be read as an estimate of the mean for the whole averages $\mathbb{E}(L_x(j,k))^q$ so that the exponent of scale is a function of log-cumulants.

3.2.2. Log-cumulants

To solve the difficulties of estimating function $\zeta(q)$ (Wendt, et al 2007) for all q propose using a polynomial for regression model . $\zeta(q) = \sum_{p\geq 1} cp \frac{q^p}{p!}$ where the coefficients c_p can be related to the cumulants of p order and satisfy $\forall p \geq 1$:

$$Cp(j) = c_{0p} + c_p \ln 2^j\ \forall p \geqslant j \tag{19}$$

With the constant c_{0p} does not play any role in fractal analysis.

We note that:

i) Using only the first two cumulants, we arrive at a good approximation of $\zeta(q)$

$\zeta(q) \simeq c_1(q) + \frac{1}{2}q^2$ et $D(h) \simeq 1 + \frac{(h-c_1)^2}{2c_2}$

ii) The definition of cumulants implies that for $p^{'} > p$ if $c_p = 0$ so $c_{p'} = 0$ (see Wendt et al., 2009, Jaffard et al 2014)

The meanings of the log-cumulants possess a certain specificity: c_1 mainly characterizes the location of the maximum of $D(h)$, c_2 corresponding to its width and c_3 corresponding to its asymmetry. Thus, all multifractal information of the signal X is contained in the triplet (c1; c2; c3).

(Kantelhardt, et al.,2002) show under certain condition of uniformity of Holder

$$S_q^L(j) \sim Fq2^{j\zeta(q)} \quad \text{with } 2^j \longrightarrow 0 \tag{20}$$

The estimator $\hat{\zeta}(q)$ of $\zeta(q)$ is obtained by a linear regression of j on $\ln_2 S_q^j$

$\hat{\tau}(q) = \sum_{j=j_1}^{j_2} \omega_j \ln_2 S_q^L(j)$.

The ponderation coefficients satisfy $\sum_{j=j_1}^{j_2} j\omega_j \equiv 1$ et $\sum_{j_1}^{j_2} \omega_j \equiv 0$

(Wendt, et al 2012) propose a parametric procedure to estimate the multifractal spectrum that writes the Legendre transform

$$\hat{f}(q) = \sum_{j=j_1}^{j_2} \omega_j \cup^L (j,q)$$

$$\hat{h}(q) = \sum_{j=j_1}^{j_2} \omega_j \vee^L (j,q)$$

Where

$$\cup^L(j,q) = \sum_{k=1}^{n_j} R_X^q(j,k) \ln_2(j,k) + \ln_2 n_j$$

$$R_X^q = L_X(j,k)^q / \sum_{k=1}^{n_j} (j,k)^q$$

The wavelet transform eliminates polynomial trend of order $N-1$.

3.3. Multifractal Detrented Fluctuation Analysis

We followed the MF-DFA procedure suggested by (Kantelhardt, et al.,2002). The process consists of five steps in which the first three steps are basically analogous to the conventional DFA procedure introduced by (Peng, et al,1994). Suppose that x_k represents a time series of finite length N with insignificant fraction of zero values and if there is any zero value exist, i.e. $x_k = 0$, it will be interpreted as having no value at k.

Stape 1: Determine the profile

$$Y(i) = \sum_{k=1}^{i}[x_k - \langle x \rangle] \tag{21}$$

where $\langle x \rangle$ denotes the mean of the entire time series.

Stape 2 Divide the profile Y(i) into $\lfloor \frac{N}{s} \rfloor$ non-overlapping segments of equal length s. Since the length N may not always be the multiple of s where some end part of the profile may remain, the same procedure is repeated starting from the opposite end of the profile so that the remaining data is not ignored. As a result, we obtained a total of 2Ns segments altogether.

Stape 3 Determine the local trend of each of the 2Ns segments using the least-square fit of the series. After that, determine the variance of each ν^{th} segment. For each segment $\nu = 1, \ldots, Ns$, the variance can be obtained by:

$$F^2(\nu, s) = \frac{1}{s}\sum_{i=1}^{s}\left\{Y\left[(\nu-1)s+i\right] - y_\nu(i)\right\}^2 \tag{22}$$

and for each segment, $\nu = 1, \ldots, N$, the variance can be found by:

$$F^2(\nu, s) = \frac{1}{s}\sum_{i=1}^{s}\left\{Y\left[N-(\nu-N_s)s+i\right] - y_\nu(i)\right\}^2 \tag{23}$$

where $y_\nu(i)$ is the fitting polynomial i.e. the local trend in the ν^{th} segment. Linear (MF-DFA1), quadratic (MF-DFA2), cubic (MF-DFA3), or higher order polynomials can be used in the fitting procedure. Since the detrending of a time series is done by subtracting the fits from the profile, different degrees of polynomial differ in their capability of eliminating trends in the series. Thus, one can estimate the type of polynomial trend in a time series by comparing the results for different detrending orders of MF-DFA (e.g. (Peng, et al,1994),(Wang, et al.,2009),(Kantelhardt, et al.,2002)).

Stape 4 Calculate the qth order fluctuation function averaging all segments $\nu = 1, \ldots, 2N_s$:

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}\left[F^2(\nu, s)\right]^{q/2}\right\}^{1/q} \tag{24}$$

where $q \in \Re$. As q approaches zero, the averaging procedure in [24] cannot be applied directly because of the diverging exponent. Therefore, the following logarithmic averaging procedure is employed as a substitute for $q = 0$:

$$F_0(s) = \exp\left\{\frac{1}{4N_s}\sum_{\nu=1}^{2N_s}\ln\left[F^2(\nu,s)\right] \sim s^{h(0)}\right\} \tag{25}$$

Note that $h(0)$ cannot be defined for times series with fractal support wher $h(q)$ diverge for $q \to 0$. The step 2 to step 5 are repeated for numerous time scale s. It is obvious that as s increases, the value of Fq(s) will increase.

Stape 5 We analyze the slope of log-log plots of $Fq(s)$ versus s for each value of q to determine the scaling behavior of fluctuation functions. The value of $Fq(s)$ will increase as a power-law for large value of s if the series x_k are long-range power-law correlated:

$$F_q(s) =\sim s^{H(q)} \tag{26}$$

Remark 3.1. *For very high value of s, $s > N/4$, $F_q(s)$ becomes imprecise due to estimation errors for small segments of size N_s. For better precision we choose $s < N/4$ with minimum value $s = 2^5$. We will often take in practice $s < 30$ to eliminate superious results $h(q)$ can be graphically analyzed by log-log-plot of $F_q(s)$ depending of s.*

The scaling exponent $H(q)$ in [26] generally may depends on q. However, in a monofractal time series, $H(q)$ is independent of q since the scaling behavior of the variances in [22] and [23] is identical for all segments ν. On the contrary, in a multifractal time series, there will be a notable dependence of $H(q)$ on q due to the different scaling behaviors in the small and large fluctuations. The scaling exponent h(q) is known as the generalized Hurst exponent seeing that $H(2)$ is identical to the well-known Hurst exponent. The relationship between the generalized Hurst exponent $H(q)$ and the classical multifractal scaling exponent $\tau(q)$ can be established using:

$$\tau(q) = qH(q) - 1 \tag{27}$$

We can determine the degree of similatritry, (Schumann and Kantelhardt, 2011), (or strength of multifractality) in finite limit [–q, +q] by:

$$\Delta H(q) = H(qmin) - H(qmax). \tag{28}$$

We can use the singularity spectrum $f(\alpha)$ to describ the data multifractal. The parameter α the hölder exponent. We can use the [28] via the Legendre transform to establish the equations:

$$\alpha = H(q) + qH'(q) \quad \text{and } f(\alpha) = q[\alpha - H(q) + 1] \tag{29}$$

Another quantify for the multifractality degree for the same limit is:

$$\Delta\alpha = |\alpha_{qmin} - \alpha_{qmax}| \tag{30}$$

4. Data and Empirical Results

4.1. DATA

We use the daily data of the BRVM10 index of the UEMOA regional stock exchange from April 17, 2000 to August 23, 2016. The data are obtained on the website of the BRVM (http: //www.brvm. org).

Comment Fig 1: represents the dynamics of price. It represents the serial evolution at the level of BRVM10 index of WAEMU stock market **(Fig a)** on the left). We note periods of decline and rise that can be explained by the periods of crises noted in Ivory Coast.**Fig b)** on the right shows the presence of clustering effect which is characterized by a grouping of extremes.

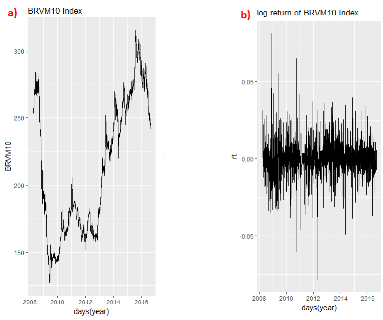

Figure 1: dynamics of BRVM10 Index and its log return

Comment: The descriptive statistics are presented in Table 1 below. The log-returns avrage is positive. The returns series is asymmetric and leptokurtic according to asymmetry coefficients>0 and kurtosis>3. These results are in line with the Jarque-Bera and Kolmogorov normality tests, which reject both the normality assumption of the distribution (P-value $=< 2.2e-16 < 5\%$).

Elements	mean	min	max	st-dev	skwness	kurtosis	jarque-Bera	Kolmogorov
rt	0.0003	-0.1210	0.1462	0.0110	0.2836	25.7891	X-squared=101 610 $(< 2, 2e-16)$	D=0,13042 $(< 2, 2e-16)$

Table 1: Descriptive statistics

4.2. EMPIRICAL RESULTS

Comment : Table 2 $0.5 > H > 1$ indicates that the process increments are positively correlated and that the process exhibits a long-term dependency and a persistent fractional Brownian motion. Our results indicate that price increases are positively correlated with each other (Joseph effect). Positive or negative trends are likely to be pursued in the same direction. And the fractal dimension that is given by $1/H$ corresponds in our case to a dimension less than 2. In other words, links of dependency

Elements	awc Hurst	FEXPMS	LD estimate	WaveBBJadaptif	genhurstw
Hurst-exponent	0.6304	0.5449	0.578,-0.422 [0.541,0.615] [-0.459,-0.385]	0.5440	0.5767

Table 2: Hurst exponent estimation

connect stock prices over time. The results indicate that small fluctuations have higher $H(q)$ than large fluctuations. The WAEMU stock market has long memory characteristics.

Comment : Table 3 shows the evolution of scale exponent, multifractal density and generalized Hurst index. The estimators of these different variables are obtained using the Discret Wavelet Transformation (DWT) and the Leaders Wavelet Transformation (LWT) method. [3] Note that the generalized index $H(q)$ varies in function of q, it decreases as q increases. In other words, the logarithmic return exhibits a multifractal process.

Moments	**ESTIMATES DWT**			**ESTIMATES LWT**		
q	$\zeta(q)$	**D(q)**	**H(q)**	$\zeta(q)$	**D(q)**	**H(q)**
-5	-6.124*** (1.852)	-0.212 (0.336)	1,467*** (0,387)	2.833*** (0.222)	0.135 (0.181)	0,74*** (0,071)
-4	-4.661*** (1.468)	-0.171 (0.336)	1,458*** (0,396)	2.123** * (0.160)	0.504*** (0.148)	0,678*** (0,064)
-3	-3.212*** (1.075)	-0.104 (0.340)	1,439*** (0,415)	1.480** * (0.107)	0.667*** (0.074)	0,604*** (0,053)
-2	-1.790*** (0.656)	-0.002 (0.374)	1,396*** (0,466)	0.915 *** (0.064)	0.861*** (0.031)	0,527*** (0,041)
-1	-0.534*** (0.206)	0.585 (0.239)	0,949*** (0,424)	0.424*** (0.029)	0.968*** (0.007)	0,456*** (0,032)
1	0.261*** (0.071)	0.970*** (0.028)	0,231*** (0,075)	0.362*** (0.029)	0.969*** (0.010)	0,332*** (0,034)
2	0.457*** (0.148)	0.853*** (0.087)	0,155* (0,094)	0.662*** (0.068)	0.871*** (0.043)	0,266*** (0,05)
3	0.569** (0.238)	0.649*** (0.140)	0,073 (0,104)	0.895*** (0.124)	0.705*** (0.096)	0,2*** (0,068)
4	0.612* (0.333)	0.467*** (0.169)	0,02 (0,101)	1.065*** (0.194)	0.504*** (0.148)	0,142* (0,079)
5	0.618 (0.423)	0.358 (0.182)	0,005 (0,094)	1.184*** (0.271)	0.314 * (0.187)	0,1* (0,084)

*Notes:(.) standard error in parentheses, * 10 percent significant, ** 5 percent significant, *** 1 percent significant*

Table 3: Hurst estimation by DWT and LWT

Comment Fig2: We can see that $\zeta(h)$ deviates from the linear behavior of a monofractal process. In both cases, $\zeta(h)$ shows a downward concavity characteristic of the multifractal processes. Equivalently,

[3] for the sake of space the table has been reduced to the interval [-5,5] but we have worked on the interval [-10,10]

$D(h)$ shows a large support that is not reduced to a single point, as indicated by the confidence intervals of the extreme points which, although broad, do not overlap. These results suggest that our analysis should instead be based on the multifractal paradigm.

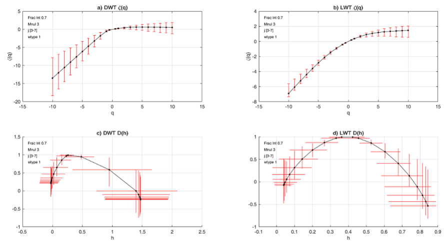

Figure 2: Scales of the law Estimation of the exponent and the Multifractal spectrum

Comment : In the Fig 3 **b)** at the top and right the value of $H(q)$ varies according to q. Like the wavelet method, the generalized Hurst exponent decreases as q increases. **d)** on the bottom right we have the multifractal spectrum and the maximum is close to 1, we have $0.5 < H < 1$ so we have a multifractal process.

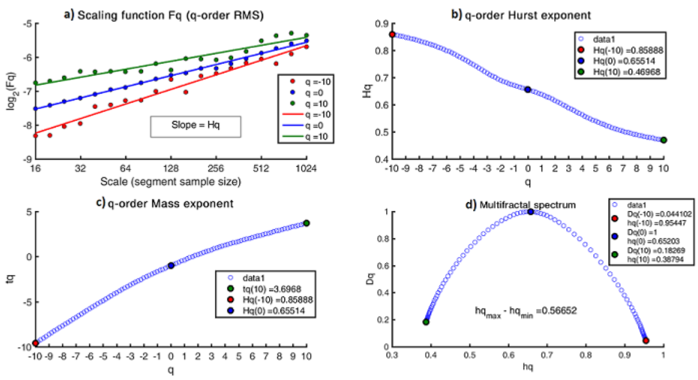

Figure 3: Multifractal analysis by MF-DFA

Comment : The figure 4 shows zeta (h) and D (h) log-return obtained with DWT and LWT. The red lines indicate the 95% confidence intervals based on the bootstrap (Wendt et al., 2002). We have a structure function for q (-2, -1, 1, 2).

Comment 5 : c1 is significant regardless of the method used. c2 is significant only with the LWT method. On the other hand, c3 is not significant whatever the method. The results of c1 and c2 confirm that the logarithmic returns follows a multifractal process.

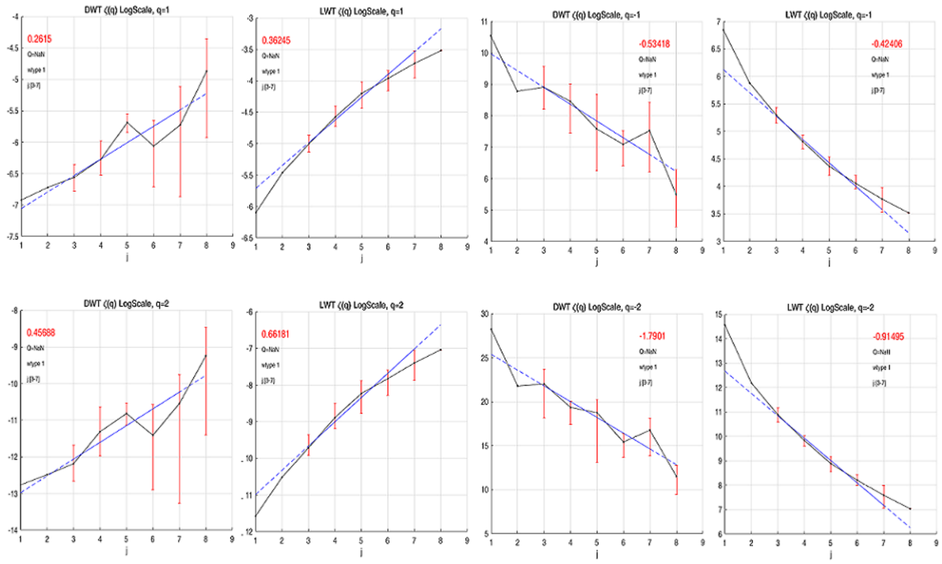

Figure 4: relation between the exponents of scale $\zeta(q)$ and the scale j

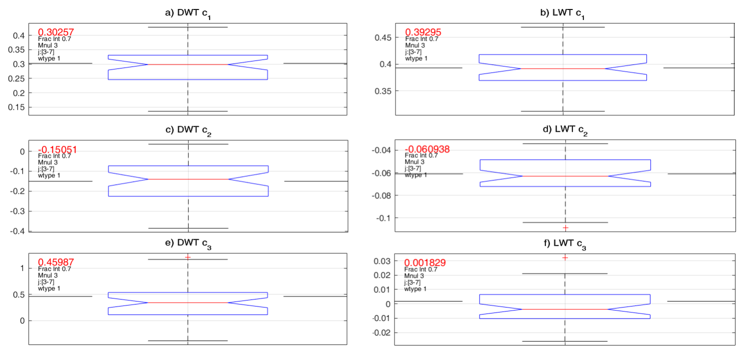

Figure 5: log-cumulant

- The first cumulant is the slope estimate, in other words, it captures the linear behavior.
- The second cumulant captures the first departure from linearity. You can think of the second cumulant as the coefficients of a second-order (quadratic) term,
- while the third cumulant characterizes a more complicated departure from the exponents of the scaling of the linearity

Comment : Fig6 We observe a very clear linear behavior for the different scales, especially the graphs obtained from LWT. The blue lines represent the linear adjustments obtained in each case, showing a perfect fit with the data for the different scales selected and confirms that the data verify the theoretical invariance properties of scale models. A theoretical requirement on multifractal models is that the

Cumulant	Estimate DWT	Estimate LWT
c1	0.303** (0.081)	0.393** * (0.028)
c2	-0.151 (0.152)	0.061 ** (0.016)
c3	0.460 (0.497)	0.002 (0.010)

*Notes: (.) standard error , * 10 percent significant, ** 10 percent significant, *** 1 percent significant*

Table 4: Log cumulant estimate

behavior of scales where the scale invariance observed must be the same for all statistical orders, i. e., q in the case of S (j, q); or p in the case of cp(j). This is indeed the case in the figure, where the scales associated with each j belongs [3: 7]. The appropriate graphs are observed for both S (j, q) and and for cp. (1), for all statistical orders and for both approaches. This last remark could have an interesting financial interpretation, as it seems to suggest that the financial mechanisms responsible for scale invariance operate at the same time.

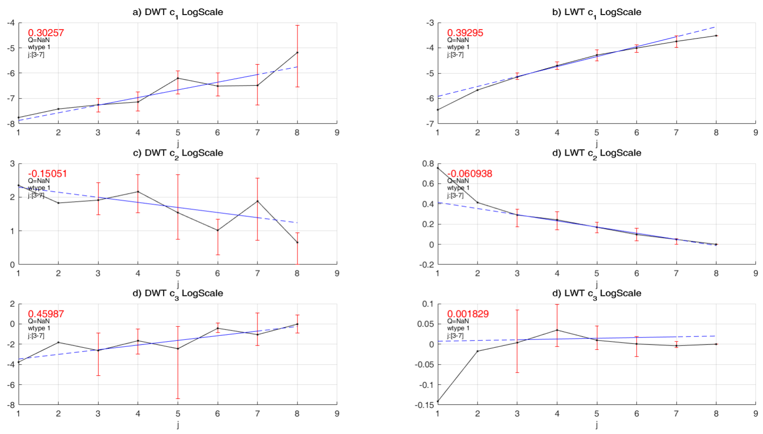

Figure 6: estimate log leaders cumulant

5. Conclusion and policy implications

This study allowed us to examine the efficient market hypothesis of the financial markets through the BRVM10 index of the regional stock exchange which groups together the 8 WAEMU countries. We used the wavelet approach and multifractal detrended fluctuations analysis. The series studied shows tails of distribution thicker than those of a normal distribution. Our results indicate that the WAEMU Regional Stock Exchange is multifractal in nature. We have a Hurst $1/2 < H < 1$, so we have a persistent process. The BRVM10 index conform to the multifractality principle of the financial markets. However, it is important to note that small fluctuations have a greater persistence dynamic. As previously mentioned, parasitic multifractality can be induced by complex non-stationarities, which often exist in the studied data, for example in the hydrological domain because of the seasonal cycle or a change of climate. In our study, multifractality is confirmed by several methods (multi-scale diagram, MF-DFA and WLMF).

The financial markets efficiency can provide a global view of the situation of a market. It would make investors attractive because of the ability of this market to direct funds towards the most productive jobs and a lack of insider trading. The BRVM performed strongly, with the BRVM10 and BRVM Composite indices respectively 94.61 and 98.04 points at start-up, compared to 219.65 and 243.06 points respectively at 31 December 2017, representing increases of 132% and 148%, respectively. In addition, the brvm has integrated the MSCI and S & P indices (source www.brvm.org). But it has experienced periods of decrease of 46% because of the fall of cocoa (between January 2016 and December 2017) following a disappointing of agricultural campaign (source www.brvm.org 02/09/18). Beside, we have an informal sector which occupies an important weight in our countries and the lack of policy of accompaniment of the private companies. Also fears about the stability and sustainability of the CFA franc that revive the debate is a brake on investment. Thus, policy makers should review whether the stock exchange respect international standards globally to put in place a reliable regulatory system. Review the level of liquidity in order to be able to cope with other stock markets both on products and the level of technology used. [4]

The institutional, legal and regulatory framework can impact on the functioning of the market (Merton, 1992; Jayasuriya, 2005). The member countries of the BRVM could even be forced to work on a possible relocation of the head office to a much more stable member country to eliminate any risk (however small) of curbing investors because of the Ivorian recurring crises. Also a relaxation of regulations and stock exchange procedures in order to attract more partner could be beneficial. Encourage other member countries to get more involved, the Ivory Coast alone holds (35 of the 45 companies listed) (sources brvm 17/01/2018).

In our future perspectives, we want to broaden this work by conducting a sector-by-sector study of the sub-regional stock market to see which sectors are the most promising. In order to determine the inefficiencies of this stock market. Also, we intend to make a comparative study between African emerging markets.

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[4] BENJAMIN NDONG "The quality of financial markets can be related to efficiency concept, which is the function of what is called the market microstructure (liquidity, transaction costs, quotation system, etc.), the organizational functioning and institutional (regulation, legal and regulatory framework, transparency etc.) "

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